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PRODUCTION PLANNING AND SCHEDULING
Aggregate Planning

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Introduction

- Aggregate Planning
  - Macroscopic decisions
  - Determine levels of workforce ("soft/flexible" capacity) and mix of products to be produced in each period over a finite horizon (e.g., the next 12 months)

Introduction

Forecast of aggregate demand for a planning horizon of T periods

Aggregate Production Plan
Determination of aggregate production and workforce levels for T-period planning horizon

Master Production Schedule
Determination of production levels by item by time period

Material Requirements Planning
Detailed timetable for production and assembly of components and subassemblies

Aggregate Units of Production

- Aggregate units of production corresponding to an “average” item
- If different types of items are produced ⇒ aggregate units in terms of weight, volume, workhours, monetary value, etc.
- Example:

(A) Model no.	(B) No. worker hours required to produce one unit	(C) Market share (%)	(D) Selling price (€)	(B)x(C)
UA32	4,8	35	325	1,68
UA35	5,2	28	390	1,456
UC12	5,4	22	450	1,188
UC48	5,8	15	510	0,87
Aggregate unit		**100**		**5,194**

Overview of Aggregate Planning

Given:

- Demand forecasts $D_1, D_2, \ldots, D_T$, for aggregate product units over planning horizon T

Determine:

- Aggregate production quantities (e.g., number of units)
- Level of resources (e.g., number of workers) required to meet forecasted demand

Overview of Aggregate Planning

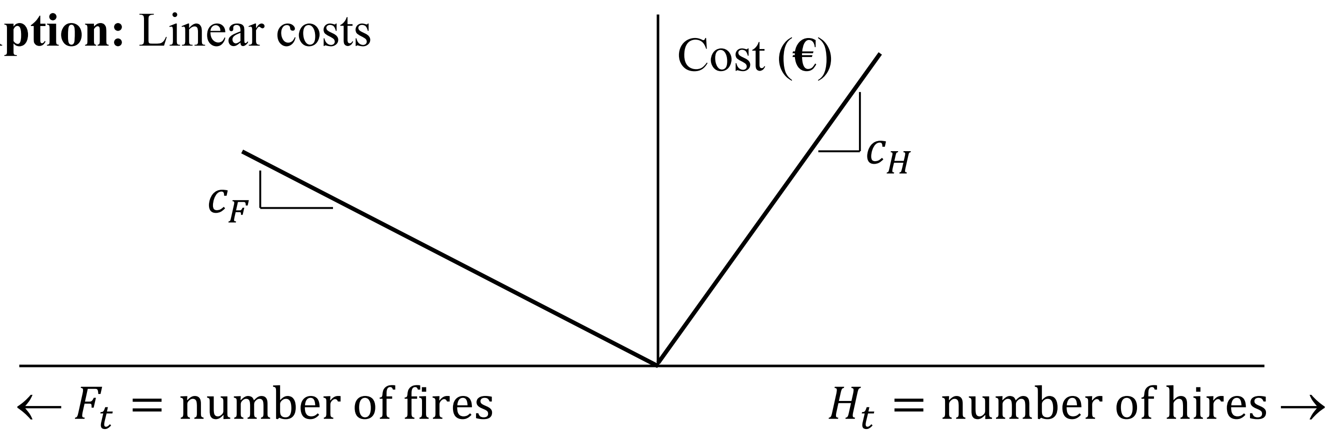
Issues

- **Smoothing**: Costs from changing production from period to period
- **Bottlenecks**: Inability to meet sudden demand changes due to capacity restrictions
- **Planning horizon**:
  - **Small *T*:** Current production levels may not be enough for meeting demand beyond the horizon
  - **Large *T***: Inaccurate forecasts for far periods
  - **End-of-horizon effect**: Use "rolling horizon"
- **Treatment of Demand**: It is assumed that demand is known with certainty (deterministic)
  - **Disadvantage**: Forecast errors
  - **Advantage**: We can incorporate seasonal fluctuations, trends, etc.

Costs in Aggregate Planning

Smoothing costs: Costs from changing production from period to period, mainly by changing workforce size

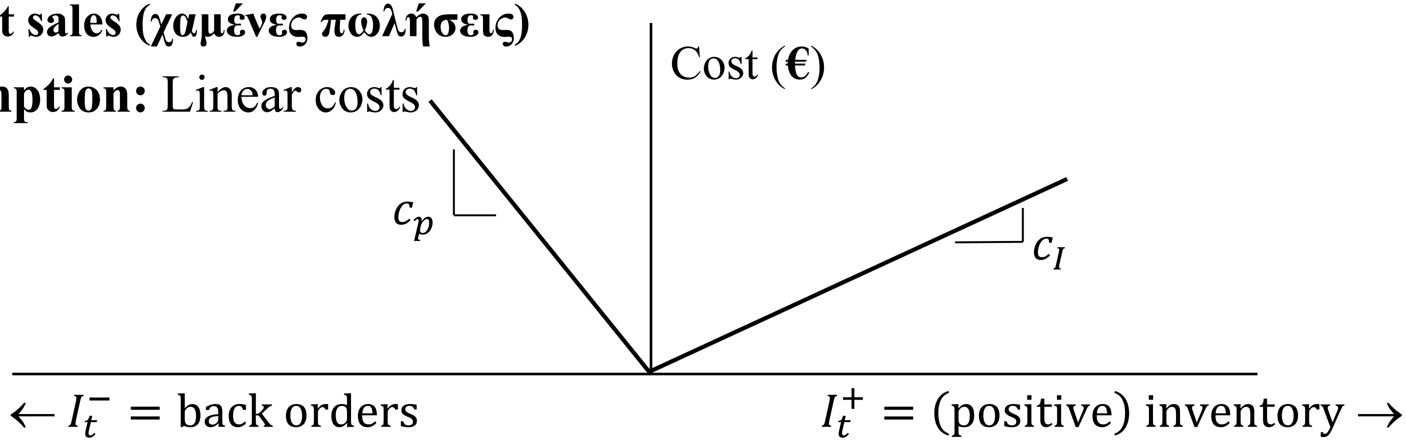
- **Increase of workforce size**:
  - time + expense to advertise position
  - Interview and screen candidates
  - Train new hires
- **Decrease of workforce size**:
  - Severance pay (αποζημίωση)
  - Decline in workforce morale (ηθικό)
  - Decrease in size of workforce pool in the future as laid off workers may find jobs elsewhere
- **Assumption:** Linear costs



Costs in Aggregate Planning

Inventory holding/shortage costs: Costs from holding inventory or not meeting demand

- **Inventory holding costs (Κόστος διατήρησης αποθέματος)**:
  - Cost of capital (money) tied up in inventories
  - Obsolescence, breakage, theft, insurance, special storage conditions, etc.
- **Shortage costs (Κόστος έλλειψης)**:
  - In manufacturing for aggregate planning purposes: Mostly **backorder cost (καθυστερημένες παραγγελίες)**
  - In highly competitive environments and in retail when managing single items: **lost sales (χαμένες πωλήσεις)**
- **Assumption:** Linear costs



Costs in Aggregate Planning

Regular time costs (κανονική απασχόληση): Costs of producing one unit of output during regular working hours.

- Payroll costs of regular employees working during regular working time
- Direct and indirect costs of materials
- Other manufacturing costs

When all production takes place in regular time, regular time costs become "sunk" costs (μη ανακτήσιμες δαπάνες) and are not affected by the planning decision.

Overtime (υπερωρία) and Subcontracting (υπεργολαβία) costs: Costs of producing one unit of output NOT during regular working hours.

- Overtime: Production of regular employees beyond normal workday
- Subcontracting: Production by an outside supplier

Idle time costs (αδράνεια): Cost of underutilization of workforce

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Competing Objectives

- **“Chase” or “zero-inventory” strategy (στρατηγική «κυνηγητού» ή «μηδενικού αποθέματος»)**: Quickly react to demand by making frequent and potentially large changes to in the size of labor force.
  - May be cost effective
  - May be poor long-run business strategy because workers who are laid off may not be available when needed
- **Stable workforce strategy (στρατηγική σταθερού εργατικού δυναμικού)**
  - May result in large buildups of inventory (manufacturing)
  - May incur substantial debt to meet payrolls in slow periods (service)
- **Maximize profits over planning horizon**

Solution of Aggregate Planning Problems by Linear Programming (LP)

Cost Parameters and Given Information

c_H = Cost of hiring one worker

c_F = Cost of firing one worker

c_I = Cost of holding one unit of stock for one period

c_R = Cost of producing one unit on regular time (κανονική απασχόληση)

c_O = Incremental cost of producing one unit on overtime (υπερωριακή απασχόληση)

c_U = Idle cost per unit of production (κόστος αδράνειας)

c_S = Cost to subcontract one unit of production

T = Number of periods in planning horizon

n_t = Number of production days in period t

K = Number of aggregate units produced by one worker in one day

I_0 = Initial inventory on hand at the start of the planning horizon

W_0 = Initial workforce at the start of the planning horizon

D_t = Forecast of demand in period t

Solution by LP

Problem Variables

W_t = Workforce level in period t

P_t = Production level in period t

I_t = Inventory level in period t

H_t = Number of workers hired in period t

F_t = Number of workers fired in period t

O_t = Overtime production in units in period t

U_t = Worker idle time ("undertime") in units in period t

S_t = Number of units subcontracted from the outside in period t

Solution by LP

LP Problem

$$\text{Minimize} \sum_{t=1}^{T} (c_H H_t + c_F F_t + c_I I_t + c_R P_t + c_O O_t + c_U U_t + c_S S_t)$$

Subject to

$$W_t = W_{t-1} + H_t - F_t, 1 \leq t \leq T \text{ (conservation of workforce)}$$

$$P_t = K n_t W_t + O_t - U_t, 1 \leq t \leq T \text{ (production \& workforce)}$$

$$I_t = I_{t-1} + P_t + S_t - D_t, 1 \leq t \leq T \text{ (inventory balance)}$$

$$H_t, F_t, I_t, O_t, U_t, S_t, W_t, P_t \geq 0 \text{ (nonnegativity)}$$

Solution by LP

Rounding the variables

Problem: W_t, H_t, F_t and often P_t should be integers.

Solution 1

Solve problem as an MILP (mixed integer LP) problem

Problem: More computational effort needed; OK for moderate-sized problems

Solution 2

Solve problem as an LP problem and round variables to nearest solution

Problem: May result in infeasible or inconsistent solution

Solution 3

Round W_t to next larger integer. Then, H_t, F_t, P_t will also be integers.

Problem: Resulting solution will rarely be optimal. Improve it by trial-and-error experimentation.

Solution by LP

Extensions

Impose minimum buffer inventory to deal with uncertainty in demand

$$I_t \geq B_t, \qquad 1 \leq t \leq T$$

Impose maximum number of hires and fires

$$H_t \leq H_t^{\max}, \qquad 1 \leq t \leq T$$

$$F_t \leq F_t^{\max} \text{ (or } F_t \leq \alpha W_t\text{)}, \qquad 1 \leq t \leq T$$

Additional capacity constraints

$$P_t \leq C_t, \qquad 1 \leq t \leq T$$

Solution by LP

Extensions

Backorders allowed

$$I_t = I_t^+ - I_t^-, \qquad 1 \le t \le T$$
$$I_t^+, I_t^- \ge 0, \qquad 1 \le t \le T$$

$$\text{Minimize} \sum_{t=1}^{T} (c_H H_t + c_F F_t + c_I I_t^+ + c_P I_t^- + c_R P_t + c_O O_t + c_U U_t + c_S S_t)$$

c_P = Cost of backordering one unit of stock for one period

Lost sales allowed

$$I_t^+ - I_t^- = I_{t-1}^+ + P_t + S_t - D_t, \qquad 1 \le t \le T \text{ (inventory balance)}$$
$$I_t^+, I_t^- \ge 0, \qquad 1 \le t \le T$$

$$\text{Minimize} \sum_{t=1}^{T} (c_H H_t + c_F F_t + c_I I_t^+ + c_L I_t^- + c_R P_t + c_O O_t + c_U U_t + c_S S_t)$$

c_L = Cost of lost sales of one unit of stock for one period

Solution by LP

Extensions

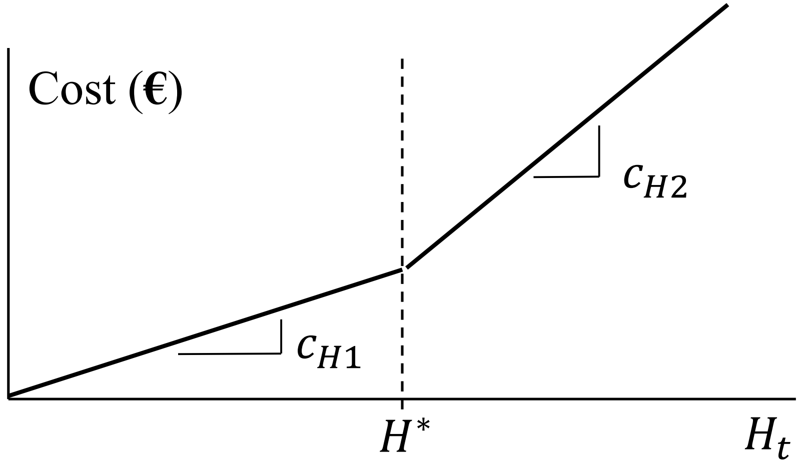
Convex piecewise-linear cost functions

$$H_t = H_{1t} + H_{2t}, \qquad 1 \le t \le T$$

$$H_{1t} \le H^*, \qquad 1 \le t \le T$$

$$H_{2t} \ge 0, \qquad 1 \le t \le T$$

$$\text{Minimize} \sum_{t=1}^{T} (c_{H1} H_{1t} + c_{H2} H_{2t})$$



Disaggregating Aggregate Plans

One possible approach

X^*= number of aggregate units of production for a particular planning period (= P_t)

Y_j= number of units of production of item $j = 1, \ldots, J$, for the same planning period

Objective function?

Inventory holding costs have already been accounted for in the determination of X^*

K_j= fixed cost of setting up for production of Y_j, $j = 1, \ldots, J$

λ_j= annual demand rate for item j, $j = 1, \ldots, J$

$$\text{Minimize} \sum_{j=1}^{J} K_j \boxed{\frac{\lambda_j}{Y_j}}$$

Measure of frequency of production

Subject to

$$\sum_{j=1}^{J} Y_j = X^* \text{ (resource allocation problem)}$$

$$a_j \leq Y_j \leq b_j, \qquad j = 1, \ldots, J$$

A Prototype Problem

Setting: A producer of disk drives wants to plan workforce and production levels for the 6-month period from January to June ($T = 6;\ t = 1, \ldots, 6$)

Number of working days and demand forecasts for a particular line of drives produced at a particular plant:

t	1	2	3	4	5	6
D_t	1280	640	900	1200	2000	1400
n_t	20	24	18	26	22	15

Initial values (at the end of December): $W_0 = 300$ workers; $I_0 = 500$ drives

Target value (at the end of June): $I_6 \geq 600$ drives

Costs: $c_H = 500$; $c_F = 1000$; $c_I = 80$

Capacity: In the past, 76 workers produced 245 disk drives over 22 working days

$$\Rightarrow K = \frac{245}{(22)(76)} = 0.14653 \text{ drives per worker per day}$$